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Group Scheduling in a Cellular Manufacturing Shop to Minimize Total Tardiness and $nT$: A Comparative Genetic Algorithm and Mathematical Modeling Approach

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Abstract

In this paper, family and job scheduling in a cellular manufacturing environment is considered. Each job is assumed to have their own individual due regardless of what family they belong to. The performance measures considered are to minimize total tardiness and the number of tardy jobs. Family splitting among cells is allowed but job splitting is not. Even though family splitting increases the number of setups and thus reduces productive time, it increases the chances of meeting individual job due dates, which often causes late delivery. Two optimization methods are employed in order to solve this problem, namely Mathematical Modeling and Genetic Algorithm (GA). The results showed that GA found the optimal solution for most of the problems tested with a high frequency. Furthermore, the proposed GA is efficient compared to the mathematical model especially for larger problems in terms of execution times. Other critical variations of problem such as family preemption only, impact of family splitting on common due date scenarios are added to the math model and finally dual objective solutions are provided and comparatively discussed. The allowance of family splitting is found to be beneficial for both common and individual due date scenarios with short to moderate setup times. The proposed comparative approach provides critical insights for the group scheduling problem in a cellular manufacturing shop with a case study.

Keywords: Cellular Manufacturing, Cell Loading, Family and Job Sequencing, Mathematical Modeling, Genetic Algorithm
1. Introduction

A Cellular Manufacturing System (CMS) is designed to produce moderate to high variety of products with low to moderate demand (Egilmez et al. 2011). CMS consists of manufacturing cell(s) with dissimilar machines needed to produce product families. Generally, the products grouped together form a product family. The benefits of CMS are lower setup, smaller lot sizes, lower work-in-process inventory and less space, reduced material handling, and shorter throughput time, simpler work flow (Suresh and Kay 1998) (Soolaki and Izadi 2013). Cell Loading deals with allocation of products to cells in a multi-cell environment. This allocation is done considering demand, processing times and due dates of the products as well as the capability and capacity of the cells (Süer et al. 1995; Suer et al. 1999). One of the critical objectives is to form product families and manufacturing cells that will work efficiently and yield the maximum productivity (Manimaran et al. 2010). Additionally, family sequencing is the task of determining the order by which product families will be processed in a particular cell. Family sequence is determined either after cell loading is completed or during cell loading process. In this paper, family splitting is allowed, i.e., some of the jobs of a family can be processed in a different cell and/or at a different point in time in the same cell (i.e., family preemption). Family setup is required and therefore each time a new family starts in a cell, a new setup is performed. Finally, Family Scheduling consists of determining start times and completion times of the product families (one segment or multiple segments) and job scheduling determines the start and completion times of individual jobs based on the family sequence established. Typically in a complex cellular system, the researchers need to address cell loading, family sequencing, family scheduling, and job scheduling tasks to obtain satisfactory results in terms of selected performance measures.

In a typical cellular manufacturing shop, the bottleneck machine is treated as single machine to solve the scheduling problem since the production rate is driven by the bottleneck machine and unit peace of flow is the main principle (Egilmez and Süer 2013). Therefore any improvement on the bottleneck machine can significantly contribute to the overall performance of the manufacturing system. In this context, any earliness or tardiness of the jobs is not desirable since earliness yields extra inventory, costs and sometimes misuse of resources, and tardiness deteriorates reputation of the company and leads to lost sales and rush shipping costs, and loss of goodwill. Earliness is especially an important problem for companies producing perishable items since the products lose some part of their shelf life before being shipped to the retailers or customers. Minimizing total earliness and total tardiness is especially important for companies working in just-in-time concept which aims to deliver the products as close to their due dates as possible (Arnold 1998). And, the impact of set-up time is yet another critical factor that needs to be addressed, when dealing with scheduling problems in manufacturing environments (Vanchipura and Sridharan 2013).

In this study, the performance measure considered is Total Tardiness (TT) and the objective is to minimize TT. If a product is completed after its due date, then it is considered a tardy product. If product is completed before its due date, then the tardiness for this product is zero (early or on-time product). As a result, the tardiness for a product takes a value of zero or positive, \( T_i = \max \{0, c_i - d_i\} \); where \( T_i \) is the tardiness for product \( (i) \), \( c_i \) is the completion time of product \( (i) \), and \( d_i \) is the due date for product \( (i) \). Even though the case study presented here is based on a shoe manufacturing company, the work done is applicable to many other cellular systems. The overall objective of this study is to solve cell loading, family and job scheduling problem in such a multi-cell environment to minimize the objectives such as total tardiness, the number of jobs. The researchers propose a mathematical model (MM) and a GA approach to solve this complex problem. An experiment is carried out using both approaches and later the results are compared. Finally, different versions of the problem are discussed and experiment results are also reported based on math model formulation.
2. Literature Review

The literature related to production control aspects of cellular manufacturing can be considered as narrow compared to cell formation in other words design aspects of cellular manufacturing. In terms of cellular control, cell loading (Suer et al., 1995), job-product sequencing and scheduling (Suer et al., 1999), manpower allocation to manufacturing cells (Egilmez et al. 2014) can be considered as the main tasks of control, which are typically applied either based on the previously formed or designed cellular shop or simultaneously. Since this paper addresses specific type of cell loading and job scheduling problem where injection molding machine (thus the operation) is the bottleneck and a cellular shop is present; only related works from the literature are aimed to be addressed. Because, cellular design and control are one of the topics that literature is abundant with works from all over the world (Egilmez et al., 2014). Several researchers worked on cell loading problem. Among these Süer et al., (1995) and Suer et al., (1999) developed initial cell loading rules to minimize total tardiness and some other measures. Besides, a few works addressed a methodology which performs cell loading and product sequencing tasks simultaneously. Süer and Dagli (2005) and Süer, Cosner, and Patten (2009) discussed models to minimize makespan, machine requirements and manpower transfers. Yarimoğlu (2009) developed a mathematical model and genetic algorithm to minimize manpower shortages in cells based on the assumption of synchronized material flow.

Since the current study focuses on job-family scheduling in other words group scheduling, it is critical to cover important works from the literature in that aspect. Regarding group scheduling-based works, most of the literature addresses machine scheduling as the problem domain where only a handful of works considers group scheduling in a cellular shop. As one of the earlier works, for instance, Nakamura et al. (1978) focused on minimizing total tardiness and considered sequence-independent family setup. Hitomi & Ham (1978) also considered sequence-independent setup times for a single machine. Ham et al. (1979) proposed a branch-and-bound algorithm for the optimal group and job sequence to minimize total flow time. Their second objective was to minimize the number of tardy jobs. Pan and Wu (1998) considered a single machine scheduling problem to minimize mean flow time subject to due date satisfaction. They have categorized the jobs into groups without considering family splitting. Ruiz and Maroto (2006) studied a hybrid flow shop scheduling problem using GA.

Liu et al. (2005) developed a GA model for a job shop scheduling problem. Gupta and Chantaravarapan (2008) studied the single machine scheduling (SMS) problem to minimize total tardiness considering group technology. Individual due dates and independent family setup times have been used in their problem with no family splitting. Nearchou (2008) studied SMS problem with common due-date jobs and developed a model using differential evolution considering earliness and tardiness as performance measures. Schaller and Gupta (2008) proposed optimal branch and bound algorithms to minimize total earliness and tardiness on a single machine scheduling problem with family setup times (Schaller and Gupta 2008). In terms of the recent works, a group scheduling problem in a two-machine flow shop with a bicriteria objective and carryover sequence-dependent setup times was studied by Yazdani et al. (2013). Additionally, case of reconfigurable manufacturing systems (Eguia et al. 2013), sequence dependent set-up times and skilled workforce assignment (Costa et al. 2014), case of robotic cells (Elmi and Topaloglu 2014) and case of batch production with minimizing the total number of changeovers (Alfieri and Nicosia 2014) are other critical works published in recent years.

In terms of works that utilized metaheuristic optimization along with mathematical optimization, Gholami and Zandieh (2008) proposed a methodology based on integrated simulation and GA model for a dynamic job shop scheduling problem. Simulation models were developed to optimize the fitness values to be used in the GA model. Hasan et al. (2011) developed a GA model with a local search technique to solve a job-shop scheduling problem considering machine breakdowns. In another work, random key alphabet was used to develop several GA models for single machine scheduling problem by Valente et al. (2010). The models penalized early and tardy jobs with quadratic costs instead of linear or maximum costs in order to minimize weighted tardiness and earliness. Models vary from each other with local search techniques, initial population generation and greedy
randomized solutions. Furthermore, Sioud et al. (2012) proposed a hybrid GA model for the SMS problem considering sequence-dependent setup times. The objective of the model was to minimize total tardiness, where the proposed approach combines GA with constraint programming, ant colony optimization, and multi-objective evolutionary algorithms at the crossover operator to improve solutions. Cheng (2012) provided three GA models for a single machine two agent scheduling problem considering learning effect. The models minimized the total completion time of the first agent without allowing tardy jobs for the second agent. Mathur and Süer (2013) used mathematical modeling and genetic algorithm for scheduling in a textile manufacturing facility considering overtime. The objective was profit maximization by minimizing the number of tardy jobs using overtime. The results showed that mathematical modeling approach was more effective than GA in most cases. Rakrouki et al. (2012) combined Genetic Local Search and Recovering Beam Search to tackle the SMS problem and provided a new heuristic called Genetic Recovering Beam Search taking the advantages of both approaches. It was proved that the new heuristic outperformed the Tabu Search and Recovering Beam Search in minimizing the total completion time on the SMS problem.

Related to the works that focus on scheduling jobs on the rotary injection molding machines in shoe manufacturing industry, Süer et al. (1999) have earlier developed a three-phase Heuristic Procedure to minimize the makespan. Subramanian (2004) has attempted to minimize the makespan for the rotary injection molding machine in the same shoe manufacturing company. He assumed the unlimited availability of the molds. Later, Urs (2005) introduced limited mold availability into the same problem to minimize makespan. Dastidar and Nagi (2005) developed a mixed integer mathematical model for injection molding scheduling with the objective of minimizing the total cost of inventory, setup and backlogging costs. Sequence-dependent setup times, multiple tooling and limited resource capacities are considered as constraints in the model. For larger problems, the authors provided a two phase decomposition method for the model in order to create solvable sub-problems. The results showed that the proposed model solves larger problems accurately within acceptable time frames. Süer et al. (2009) extended the problem with some heuristic procedures and mathematical models. Huang et al. (2011) proposed a sequential GA model for rotary machine scheduling considering sequence dependent processing and setup times. In the most recent work, Eğilmez and Süer (2011) proposed a non-linear mathematical model to the stochastic version of the problem, where group of jobs are scheduled on a rotary machine, which is considered to be a single machine scheduling problem with uncertain processing times and deterministic due dates. In this paper, cell loading and group scheduling problem are considered together in a cellular manufacturing shop. Two objectives are considered to be tackled, namely: the number of tardy jobs (nT) and total tardiness (TT). Mese’s mathematical model and a GA are comparatively experimented on various problem sizes considering single and hybrid objectives, job splitting and common due date scenarios to increase the overall understanding about the group scheduling problem in a cellular shop. The rest of the paper is organized as follows. Section 3 provides the description of the problem studied and assumptions made. Methodology is explained in section 4. Results of the experimental study are revealed in section 5. Section 6 introduces the concluding remarks and future dimensions of the current research.

3. The Problem Studied

This problem was observed in a shoe manufacturing shop where up to twelve product families exist (Mese 2009). The manufacturing shop consists of multiple cells in which the rotary injection molding machine is the bottleneck. Products are differentiated by five attributes; gender, size, sole type, color, and material. Each manufacturing cell includes lasting, rotary molding machine, and finishing/packing operations as shown in Figure 1. Lasting prepare the shoes for RMM where the materials are injected into the molds. Finishing/Packing basically removes the extra materials from the injected shoes, finish and pack the shoes.
In terms of the product family grouping, a representation code is used to identify the product families. In the 'MC' code form: M denotes the Material (PU: U, PVC: P, TPR: T), and C denotes the Color (Black: B, Dark Green: G, Honey: H, Nicotine: N). There are 12 product families (= 4 colors * 3 material types). All sizes of a specific order (with the same Model ID, Gender, Sole Type, Material, Color, and Due Date) is considered as a job. Different sizes of a job can have different demand. All of the sizes included in a job are assumed to have the same due date. In terms of other assumptions, the molds used in the Rotary Molding Machine for injection molding vary by size, gender, and sole type. It is assumed that there is not any restriction on the availability of molds. Therefore, the same size pairs of a job can be run on all positions of the Rotary Molding Machine simultaneously. In this study, setup times between jobs in the same family are assumed negligible. However, setup times (for material or color or both changes) between families are assumed to take 20 minutes. An example of customer orders that consists of 5 jobs and corresponding families is presented in Table 1.

Table 1

Example family formation

<table>
<thead>
<tr>
<th>Job ID</th>
<th>Model ID</th>
<th>Gender</th>
<th>Sole Type</th>
<th>Material</th>
<th>Color</th>
<th>Size</th>
<th>Code</th>
<th>Total Demand</th>
<th>Due Date</th>
<th>Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>M</td>
<td>FS</td>
<td>TPR</td>
<td>Red</td>
<td>5, 6, 7, ..., 15</td>
<td>TB</td>
<td>208</td>
<td>11</td>
<td>F1</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>F</td>
<td>MS</td>
<td>PU</td>
<td>Red</td>
<td>5, 6, 7, ..., 12</td>
<td>UB</td>
<td>881</td>
<td>20</td>
<td>F3</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>M</td>
<td>MS</td>
<td>PU</td>
<td>Black</td>
<td>5, 6, 7, ..., 15</td>
<td>UB</td>
<td>831</td>
<td>17</td>
<td>F3</td>
</tr>
<tr>
<td>4</td>
<td>T</td>
<td>M</td>
<td>MS</td>
<td>PU</td>
<td>Black</td>
<td>5, 6, 7, ..., 15</td>
<td>UB</td>
<td>277</td>
<td>13</td>
<td>F3</td>
</tr>
<tr>
<td>5</td>
<td>O</td>
<td>F</td>
<td>FS</td>
<td>PVC</td>
<td>Dark Green</td>
<td>5, 6, 7, ..., 12</td>
<td>PG</td>
<td>250</td>
<td>15</td>
<td>F2</td>
</tr>
</tbody>
</table>

The most important feature of the scheduling problem studied in this paper is the presence of individual due dates for each job in a family. The reason is that customers often order different amount of lots depending on varying sizes, colors, etc. which creates the case of individual due dates for each job in a job family. Ideally, the demands for certain products are grouped into families based on their processing similarity. It is desirable to run the entire family at once to take the advantage of a common setup time and also processing similarities. However, when products in a family have different due dates due to certain reasons, the tradeoff between meeting the due dates for all jobs and working with minimum amount of setup becomes a challenging task for the planner. In fact, this problem has been observed in a shoe manufacturing company during Dr. Suer’s visits as consultant. There is a natural conflict between meeting due dates of jobs versus reducing the total setup time between families. When all jobs in a family are scheduled all together, setup times are reduced. However, this may also lead to other jobs in the following families to be delayed and increases the possibility of having more tardy jobs. On the other hand, when a family is split several times, the number of setups increases thus reducing the productive time and hence may adversely affect the total tardiness in the long run. The researchers attempt to find a balance between family splitting and meeting due dates such that the total tardiness is minimized. The
nT and Tmax objectives are separately addressed by Süer & Mese (2011) and (Süer et al. 2014). In this paper, the researchers address the total tardiness objective and also discuss two other variations from the basic problem including common due date and family splitting issues. Finally, the relations between TT and nT objectives are also briefly explored with additional experiments to explore the impact of different objectives on the tradeoff between setup and delivery lateness.

4. Methodology

In this paper, both mathematical model (MM) and genetic algorithm (GA) approaches are proposed to solve the proposed problem. Therefore, the methodology is organized as two sections. First section describes the proposed mathematical model for the simultaneous cell loading, family and job scheduling to minimize total tardiness and nT. Since the proposed problem has such features as inclusion of multi objectives and dealing with job-families during optimizing the schedules for cells, mathematical optimization can only provide optimal solutions for a small-size of problems. On the other hand, it is critical to develop decision support frameworks for manufacturing cell scheduling where alternative methodologies such as metaheuristics could be used for the same objective. Therefore, GA is utilized as benchmark approach for MM, which is compared with MM for a set of problems. The hierarchical framework of the methodology is illustrated in Figure 2.

Fig. 2. Hierarchical Framework of the Proposed Methodology

4.1. Mathematical Model (MM)

This section describes the MM developed, which is a modified version of the model proposed by Süer & Mese (2011). The proposed model consists of four indices for family, job, position and cell and six decision variables and eight parameters. The notation is described as follows.

Indices:

\[ i \quad \text{Family index} \]
\[ j \quad \text{Job index} \]
\[ k \quad \text{Position index} \]
\[ m \quad \text{Cell index} \]

**Parameters**

\[ n \quad \text{Number of jobs} \]
\[ n_i \quad \text{Number of jobs in family } i \]
\[ f \quad \text{Number of families} \]
\[ M \quad \text{Number of cells} \]
\[ P_{ij} \quad \text{Process time of job } j \text{ from family } i \]
\[ D_{ij} \quad \text{Due date of job } j \text{ from family } i \]
\[ S \quad \text{Setup Time} \]
\[ R \quad \text{Big integer} \]

**Decision Variables**

\[ Y_{mk} \quad 0 \text{ if } k^{th} \text{ position in cell } m \text{ is occupied, } 1 \text{ otherwise.} \]
\[ X_{ijmk} \quad 1 \text{ if job } j \text{ from family } i \text{ is assigned to the } k^{th} \text{ position in cell } m, 0 \text{ otherwise.} \]
\[ C_{mk} \quad \text{Completion time of the job in } k^{th} \text{ position in cell } m \]
\[ T_{mk} \quad \text{Tardiness value of the job in } k^{th} \text{ position in cell } m \]
\[ nT_{mk} \quad 1 \text{ if job in } k^{th} \text{ position in cell } m \text{ is tardy, } 0 \text{ otherwise} \]
\[ W_{mk} \quad 1 \text{ if setup is needed before the job in } k^{th} \text{ position in cell } m, 0 \text{ otherwise.} \]

**Objective Function:**

\[
\min Z_{nT} = \sum_{m=1}^{M} \sum_{k=1}^{n} nT_{mk} \tag{1}
\]

\[
\min Z_{TT} = \sum_{m=1}^{M} \sum_{k=1}^{n} T_{mk} \tag{2}
\]

Subject to:

\[
\sum_{m=1}^{M} \sum_{k=1}^{n} X_{ijmk} = 1 \text{ for } i = 1, \ldots, f \text{ and } j = 1, \ldots, n_i \tag{3}
\]
\[
\sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_k} \leq 1 \quad \text{for } m = 1, , M \quad k = 1, , n
\] (4)

\[
\sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_k} \geq \sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_{k+1}} \quad \text{for } m = 1, , M \quad k = 1, , n - 1
\] (5)

\[
1 + W_{mk} \geq \sum_{j=1}^{n_i} X_{ijm_k} + \sum_{j=1}^{n_q} \sum_{q \in (f' \backslash i)} X_{ijm_{k-1}} \quad \text{for } m = 1, , M \quad k = 2, , n
\] (6)

\[
\sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_k} \leq R \cdot (1 - Y_{mk}) \quad \text{for } m = 1, , M \quad k = 1, , n
\] (7)

\[
-C_{m1} + \sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_1} \cdot P_{ij} \leq R \cdot Y_{m1} \quad m = 1, , M
\] (8.a)

\[
C_{m(k-1)} - C_{mk} + S \cdot W_{mk} + \sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_k} \cdot P_{ij} \leq R \cdot Y_{mk}
\] for \( m = 1, , M \quad k = 2, , n \) (8.b)

\[
C_{mk} - \sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijm_k} \cdot D_{ij} \leq T_{mk} \quad \text{for } m = 1, , M \quad k = 1, , n
\] (9)

\[
T_{mk} \leq R \cdot nT_{mk} \quad \text{for } m = 1, , M \quad k = 2, , n
\] (10)

Definition of Variables:

\( X_{ijm_k} \in \{0, 1\}, \quad W_{mk} \in \{0, 1\}, \quad Y_{mk} \in \{0, 1\}, \)

\( C_{mk} \geq 0, \quad T_{mk} \geq 0 \)

The objective functions are to minimize the number of tardy jobs (nT) and Total Tardiness (TT) and given in Equations (1) and (2). For single objective problems, equations 1 and 2 are used individually. For the multi-
objective cases, both equations are used together. According to Equations (3), each job can be assigned only once. Equation (4) guarantees that each position in each cell can be assigned at most one job. Equation (5) enforces jobs to be assigned consecutively in each cell. Equation (6) deals with setup requirements. If the consecutive jobs come from different families, then this constraint guarantees that a setup is added between those consecutive jobs. In Equations (7), (8.a) and (8.b), If-then constraints are used to eliminate the nonlinearity in the model. Equation (7) checks if a position is occupied by a job. If so, Equations (8.a) and (8.b) determine the completion time of the job in that position. Equation (9) calculates the tardiness value of a job and Equation (10) performs as the counter for determining the number of tardy jobs ($n_T$).

4.2. Genetic Algorithm

The proposed MM cannot solve large problems due to computational requirements. As a result, a genetic algorithm approach is proposed and later its performance is measured against the math model results. First, the initial population of n chromosomes is formed randomly. Then, mating partners are determined using mating strategies to perform crossover. The crossover and mutation operators are performed to generate offspring. For selecting the next generation, parents are added to the selection pool along with offspring. The next generation is selected from this pool based on their fitness function value. These steps are repeated until the number of the generations specified by the user is reached. Finally, the best chromosome obtained during the entire evolutionary process is taken as the final solution. Following is the brief list of notation used in GA:

- $G$: Number of Generations
- $PS$: Population Size
- $PC$: Crossover Probability
- $PE$: Elite ratio
- $PW$: Worst ratio
- $PMJ$: Mutation Probability for Jobs
- $PMC$: Mutation Probability for Cells

4.2.1. Chromosome Representation

Each chromosome consists of n genes and each gene corresponds to a job. For each gene, code $(X, Y)$ is used where $X$ denotes the job number and $Y$ denotes the cell it is assigned. The sequence of genes in a chromosome also establishes the sequence of jobs in the cells. An illustration is given in Figure 3, where the sequence of jobs in cell 1 is Jobs 2, 1 and 6, in cell 2: it is Jobs 5, 3 and 4 and similarly, in cell 3, it is Jobs 8, 7, 9.

![Fig. 3. A Chromosome Representation](image)

4.2.2. Mating

Three mating strategies are used; 1) Random (R), 2) Best-Best (B-B), and 3) Best-Worst (B-W). The reproduction probabilities of the chromosomes are calculated according to their fitness function. The next step depends on the mating strategy used.
1) Random Mating Strategy; Each chromosome in the population is mated with a randomly selected partner and they produce one offspring. The partner is selected using reproduction probability based on Roulette Wheel approach. By using the Random Mating Strategy, \( PS \) mating pairs are determined to generate \( PS \) offspring.

2) Best-Best Mating Strategy; All chromosomes are ranked with respect to their reproduction probabilities (in descending order). Then, the top two chromosomes form a pair, the next top two chromosomes form another pair and so on. In addition, the first \( X\% \) of the pairs produce 3 offspring, the next \( Y\% \) of the pairs produce 2 offspring, and the remaining pairs produce 1 offspring. By the Best-Best Mating Strategy, \( (PS/2) \) mating pairs are determined to generate at least \( PS \) offspring.

3) Best-Worst Mating Strategy; After chromosomes are ranked as in B-B strategy, the best chromosome is paired with the worst chromosome; the second best chromosome is paired with the second worst chromosome and so on. All pairs produce 2 offspring. By the Best-Worst Mating Strategy, \( (PS/2) \) mating pairs are determined to generate at least \( PS \) offspring.

4.3.3. Crossover

Two different strategies are used; 1) Position-Based Crossover (P-B) and, 2) Order Crossover (OX) Strategies (Gen & Cheng (1997). The crossover operation is applied to the identified pairs with a probability of \( P_c \). The first parent is copied ‘as is’ as the offspring if crossover is not performed. One important point is that the crossover operator affects only the sequence of jobs. In other words, the crossover is applied only to the genes’ X element and not to Y element. Following is the explanation of the aforementioned crossover procedures undertaken:

- After determining the mating pairs that will go through the crossover step, the crossover is executed according to the selected crossover strategy. If the Position-Based Crossover Strategy is selected, following steps are applied: First, a set of genes are selected from the first parent with a probability of 0.5. Second, the X values of the selected genes are copied to the corresponding genes of the produced offspring. Third, the selected X values are deleted from the second parent. Finally, the X values left in the parent chromosome after deleting are placed into the unfixed genes of the offspring from left to right according to the order of the genes to produce the offspring completely.

- If the Order Crossover Strategy is selected, following steps are applied: First two genes are selected with a probability of 0.5 from the first parent to determine the first and the last genes of a substring. Second, the X values of this substring are copied into the corresponding genes of a chromosome in order to produce an offspring. Third, the selected X values are deleted from the second parent. Finally, the X values left in the parent chromosome after deleting are placed into the unfixed genes of the offspring from left to right according to the order of the genes to produce the offspring completely. This procedure is followed until all offspring are produced. After crossover operations, all offspring go through the mutation step.

4.4.4. Mutation

Two steps are used in the mutation operator. The first one is used for job sequence and only Reciprocal Exchange (R-E) Mutation Strategy is used (Gen & Cheng (1997). In R-E Mutation strategy, the offspring, after randomly being selected, is mutated by swapping two randomly selected X genes of the chromosome. The mutation for job sequence is performed with a probability of \( P_M \). The second step involves mutating cell assignments (e.g., only Y gene is mutated). In the mutation of cell assignments, two different mutation strategies are used, namely: Random (R), and Reciprocal Exchange Mutation. The mutation of the cell assignment is performed with a probability of \( P_{MC} \). In Random Mutation strategy, the offspring is mutated by randomly
selecting a set of genes, and changing the Y values of the genes randomly with all cell numbers having equal probability. After mutation operations, all offspring are transferred to the selection pool for the next generation.

### 4.4.5. Selection

In this study, selection pool consists of all offspring and some of the parents. The next generation is selected from this pool. The selection from parents is a two-step process. First, the parents are ranked with respect to their reproduction probability in descending order. Then some of the parents are from the top of the list are designated as elites and advanced to the selection pool. The percentage of the selected parents from PS is found using a ratio called elite ratio (PE*PS). Next, the best PE% parents are directly selected to advance to the selection pool. Then, the remaining (100-PE)% chromosomes are selected from the parents randomly based on their reproduction probability using Roulette Wheel Selection. Once the selection pool is identified, the chromosomes are ranked with respect to their reproduction probability and a final selection is made from this pool to generate the next generation. In some experiments, the researchers also allowed a certain percentage of lowest performers (Pw%) to advance automatically to the next generation to avoid immature convergence of the population. The percentage of the lowest performing parents from PS is found using a ratio called worst ratio (PW*PS). All selected chromosomes constitute the next generation. This procedure is repeated until the number of the generations reaches to a specified number. The best chromosome among all generations is set as the best solution of GA model.

### 5. Results


#### 5.1. Experiments with GA

In this experiment, three datasets were used to determine the best GA parameters. The experiment started with a set of default values and then the values of the GA parameters are changed one at a time in order to obtain better combinations. The list of combinations was summarized in Table 2.

#### Table 2

<table>
<thead>
<tr>
<th>Elite R</th>
<th>Worst R</th>
<th>p_c</th>
<th>p_mj</th>
<th>p_mc</th>
<th>Mating Strategy</th>
<th>Crossover Strategy</th>
<th>Mutation Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Combination 1</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>R</td>
<td>P-B</td>
</tr>
<tr>
<td><strong>Combination 2</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>B-B</td>
<td>P-B</td>
</tr>
<tr>
<td><strong>Combination 3</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>B-W</td>
<td>P-B</td>
</tr>
<tr>
<td><strong>Combination 4</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>R</td>
<td>OX</td>
</tr>
<tr>
<td><strong>Combination 5</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>B-B</td>
<td>OX</td>
</tr>
<tr>
<td><strong>Combination 6</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.7</td>
<td>0.05</td>
<td>0.5</td>
<td>B-Wt</td>
<td>OX</td>
</tr>
</tbody>
</table>
5.2. Comparison of Math Models & GA

In this section, the results of the MMs are compared with the GA results. The experimental conditions that are detailed in the previous section are utilized in GA runs. The MM solutions (optimal solutions) and GA solutions for configurations are given in Table 3 and Table 4. Solutions with single cell include configurations 1-3 and shown in Table 3. Multi cell solutions of configurations 4-6 are provided in Table 4.

### Table 3

Results for Minimizing TT for One Cell

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Math Model</th>
<th>The Number of Decision Variables &amp; Constraints</th>
<th>Math Model Execution Time (hr:min:sec)</th>
<th>Optimal Frequency for GA (x/10)</th>
<th>GA Execution Time (hr:min:sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1107.2</td>
<td>155 - 95</td>
<td>00:00:02</td>
<td>10</td>
<td>00:00:59</td>
</tr>
<tr>
<td>2</td>
<td>1611.7</td>
<td>181 – 148</td>
<td>00:00:40</td>
<td>10</td>
<td>00:01:01</td>
</tr>
<tr>
<td>3</td>
<td>743</td>
<td>209 - 161</td>
<td>00:00:19</td>
<td>10</td>
<td>00:01:06</td>
</tr>
</tbody>
</table>

GA found the optimal solution ten times out of ten replications for one cell problem and with a high frequency for multiple cells case. Execution times for Math Model and GA were not significantly different for one cell problems whereas the gap increased significantly for multi-cell problems.

### Table 4

Results for Minimizing TT for Multiple Cells

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Math Model</th>
<th>The Number of Decision Variables &amp; Constraints</th>
<th>Math Model Execution Time (hr:min:sec)</th>
<th>Optimal Freq. for GA (x/10)</th>
<th>GA Exe. Time (hr:min:sec)</th>
<th>Avg. of GA Results (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>345.6</td>
<td>854 – 406</td>
<td>00:25:20</td>
<td>9</td>
<td>00:01:16</td>
<td>347.6</td>
</tr>
<tr>
<td>5</td>
<td>1162.4</td>
<td>854 – 490</td>
<td>11:43:38</td>
<td>7</td>
<td>00:01:19</td>
<td>1164.8</td>
</tr>
<tr>
<td>6</td>
<td>432.4</td>
<td>1426 – 674</td>
<td>11:21:14</td>
<td>9</td>
<td>00:01:41</td>
<td>434.45</td>
</tr>
</tbody>
</table>

5.3. No Family Splitting for Single Cell Case

The approach proposed by Gupta & Chantaravarapan (2008b) are run with five datasets that have varying family and job configurations. The results of configurations 7-11 are shown in Table 5. Family splitting (FS) resulted in lower total tardiness values in all datasets. Based on the results, it can be concluded that allowing family splitting will produce lower total tardiness values (equal results in worst case scenario). The results show
that families were split several times and this led to better results. However, the length of setup times will affect
the number of splits. The impact of setup times is left as a future work.

Table 5
The impact of Family Splitting (FS)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>The number of families</th>
<th>The number of jobs</th>
<th>Configuration</th>
<th>Total Tardiness</th>
<th>Number of Setups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>FS is NOT Allowed</td>
<td>FS is Allowed</td>
<td>FS is NOT Allowed</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>7</td>
<td>3-4</td>
<td>1084</td>
<td>896</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>9</td>
<td>3-3-3</td>
<td>1610.3</td>
<td>993.2</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>10</td>
<td>2-2-3-3</td>
<td>4264.2</td>
<td>3866.7</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>12</td>
<td>3-3-3-3</td>
<td>5075.6</td>
<td>2994.5</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>12</td>
<td>3-3-3-3</td>
<td>3807.5</td>
<td>2034.6</td>
</tr>
</tbody>
</table>

5.4. Family Preemption Only (FPO) Strategy

In this section, family splitting is allowed however, it is restricted to only one cell. This strategy is called as
“Family Preemption Only”. In some manufacturing systems, family splitting among cells is not desirable due to
setup restrictions. This strategy can be used in such circumstances to schedule each family to only one cell with
allowing family splitting within the cell, which can be also the case in food production environments where
certain group of food products need to be dedicated to certain cells due to hygiene-related or health reasons
(Egilmez et al. 2012). The results of experimentation performed with three datasets are shown in Table 6.
According to the results, this constraint adversely affects the total tardiness values. The Gantt chart for
configuration 4 is illustrated in Figure 4. It can be concluded that, as long as there is no restriction, the allowance
of family splitting among cells can result in better schedules than within one cell.

Table 6
Comparison of FS vs. FPO Strategies

<table>
<thead>
<tr>
<th>Configuration</th>
<th>The number of families</th>
<th>The number of jobs</th>
<th>FS</th>
<th>FPO</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>15</td>
<td>345.6</td>
<td>1008.7</td>
</tr>
</tbody>
</table>
5.5. The Impact of Family Splitting on Common Due Date

In this section, the impact of family splitting on common due date is analyzed. Gupta & Chantaravarapan (2008b) considered common due dates and no family splitting for the single machine problem. In this study, it is considered that family splitting among manufacturing cells is significant where jobs have individual due dates. Moreover, family splitting can be beneficial where jobs from the same family have same due date (common due date). To illustrate the case, an 8-job 2-family problem is derived. It is assumed that all jobs in a family has common due date. The processing time and due date information is given in Table 7. The example problem is solved based on the two objectives: nT and TT. The results are provided as Gantt charts in Table 8 (nT) and Table 9 (TT).

Fig. 4. Comparison of Family Splitting and Family Preemption Only Strategies

<table>
<thead>
<tr>
<th>Table 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example problem data</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Family</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
</table>
According to the results of nT objective in Table 8, family splitting allowance resulted in 2 tardy jobs whereas 3 tardy jobs are obtained when family splitting is not allowed. In terms of TT, family splitting allowance resulted in total tardiness of 363 where total tardiness of 522 is obtained when family splitting is not allowed (see Table 9). In conclusion, allowance of family splitting can also be beneficial in common due date problem based on both nT and TT objectives.
5.6. Dual Objectives

Finally, both of the performance measures (total tardiness, TT and number of tardy jobs, nT) are simultaneously considered in this section. A bi-criteria model is developed with a minor modification on the proposed model. Both single objective (nT and TT) and dual objective models (nT(1)-TT(2) and TT(1)-nT(2)) are run and the results are compared for three datasets. The details of datasets are given in Table 10 along with results. In the dual models, the primary objective decision variables are treated the same however, the secondary objective decision variables are multiplied with a factor. The factors have been determined after some trial runs so that the optimal solution is found with respect to primary objective and the secondary performance measure improves. The configurations and results of datasets are shown in tables 10 and 11, respectively. When dual measures were used, the total tardiness improved significantly when used as a secondary measure.

Table 10
Configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Number of jobs</th>
<th>Number of families</th>
<th>Number of cells</th>
<th>Formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>3</td>
<td>3</td>
<td>3-3-3</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>2-2-2-2</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>4</td>
<td>3</td>
<td>3-3-3-3</td>
</tr>
</tbody>
</table>

Table 11
Dual Objective vs. Single Objective

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Dual</th>
<th>Single</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT</td>
<td>TT</td>
<td>nT</td>
<td>TT</td>
<td>nT</td>
</tr>
<tr>
<td>nT</td>
<td>(nT)</td>
<td>TT</td>
<td>nT</td>
<td>(TT)</td>
</tr>
<tr>
<td>325.2</td>
<td>3</td>
<td>325.2</td>
<td>3</td>
<td>1368.5</td>
</tr>
<tr>
<td>292.2</td>
<td>4</td>
<td>292.2</td>
<td>4</td>
<td>878.6</td>
</tr>
<tr>
<td>866.5</td>
<td>5</td>
<td>866.5</td>
<td>5</td>
<td>1275.5</td>
</tr>
</tbody>
</table>

6. Conclusions, Managerial Implications and Future Work

In this paper, multi objective cell, family and job sequencing problem is studied considering two performance measures, namely: minimizing the number of tardy jobs and total tardiness. Every job is assumed to have individual due date in contrast to common family due date concept. Each family requires a setup before the jobs in that family can be processed. This creates a natural conflict between meeting due dates of jobs and reducing total setup times. If the entire family is scheduled together, then the total setup time will be the minimum. But, the jobs in the consecutive families may be postponed and probably total tardiness will increase. In contrast, splitting a family several times may increase the number of setups which reduce the productive time, and finally have an adverse effect on total tardiness.

Mathematical modeling is one of the available and reliable solution techniques, which guarantees to find the optimal solution. However, it often becomes a challenging task to solve large problems using MMs because of
the computational requirements and experiment times. As a result, there is a need to use other approaches to solve such problems. Genetic Algorithm (GA) approach is proposed as alternative solution approach to larger size problems. GA found the optimal solution in all problems with high frequency. The execution time of MM was reasonable only for small problem sizes. GA clearly outperformed MM with respect to execution times.

The results showed that family splitting occurred in all multi-cell problems. The allowance of family splitting (FSA) resulted in better schedules in terms of both $nT$ and $TT$. The occurrence of family splitting in these problems show us that the system used the feature of family splitting since it was beneficial in terms of reducing total tardiness. In some industries, managers may prefer this option to control and deal with quality issues and also due to learning effect on the workers. Therefore, another possible variation from the basic problem is considered; family preemption only (FPO). The main idea behind this strategy was to allow family splitting but limit each family to a single cell. Finally, bi-criteria cell loading, family and job sequencing is also experimented.

The unique contributions of this paper, related managerial implications and limitations are summarized as follows:

- The solution procedure provides flexibility with dealing with multiple objectives (single or hybrid objective cases for minimizing $nT$ and $TT$). Operations people always deal with different customers with varying expectations. Depending on the customer expectation level and type of partnership, scheduling task could be handled considering single focus on minimizing only the number of tardy jobs or the total tardiness, or both simultaneously.

- This paper enhances group scheduling on manufacturing cells considering various extended options for schedulers such as allowance of family splitting, family preemption and common due date options. These options touch to the real situations that operations people deal with in real industries. For instance, even though it is not always a desired concept, family splitting can be critical if the jobs in a family are related with individual customers with different expectations and levels of business relationships. Additionally, family preemption will be beneficial and can improve the productivity significantly, if individual job-families are defined based on the product type which is dedicated to an individual cell. For instance, preemption strategy can work on jobs where product A’s orders from different customers are considered to be met by cell A, then product B’s orders by cell B. In terms of the common due date, it’s a robust planning option where operations scheduler can group orders for different products from the same customer based on assigning a common delivery date.

- In terms of the alternative solution procedure, GA provided reliable performance in most cases compared to the optimal solution provided by MM. The experimentation times were significantly reduced by GA, which can help decision making process to be faster. Since scheduling decisions are made in a highly dynamic manufacturing environment and revised on a daily basis, it is critical to have the heuristic methods that could assist with scheduling related decision making in faster time with reliably accuracy.

- The main limitation of the current research is the proposed methodology’s deterministic characteristics. Processing times and due dates are considered to be deterministic, which is ideal if the manufacturing environment is machine-oriented. However, in labor oriented manufacturing shops, labor skills cause variation in processing times and assuming due dates uncertain or to follow a probability distribution would make the problem more complex and challenging but enable practitioners provide more robust solutions that better deal with uncertainty. In this regard, stochastic programming models would be a critical future dimension, which was previously utilized in works such as (Egilmez et al. 2011; Egilmez et al. 2012; Egilmez & Süer 2011; Egilmez et al. 2014).

The future directions of current research include consideration of sequence-dependent setup times, job splitting and experimentation with other meta-heuristic techniques. In addition, the current scheduling problem can also be studied from an inverse scheduling perspective where a job sequence is given and the objective is to
determine the minimal perturbation to the job (Koulamas 2005) and connection with the layout aspects can be considered in parallel with Ariafar et al. (2012) and Manzini et al. (2006). Finally, alternative family formations can be experimented and the efficiency of schedules could be compared by using Data Envelopment Analysis similar to recent works such as Aalaei et al. (2014), Pourjavad & Shirouyehzad (2014).

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**References**


